

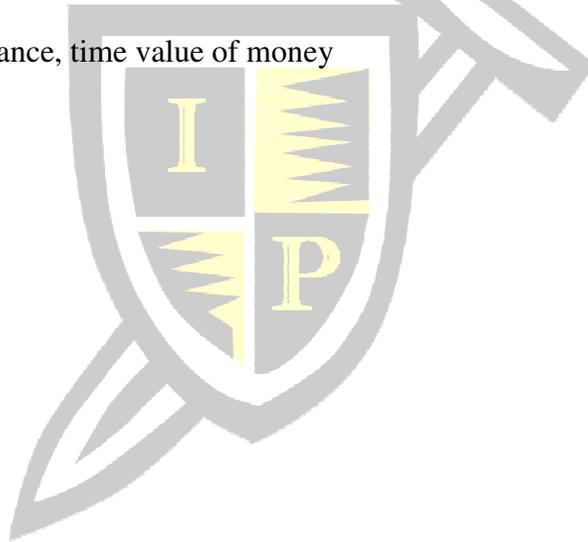
## Teaching time value of money to dyslexic students: a pilot case study

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### ABSTRACT

This essay advocates for and investigates the usefulness of a modified version of the Jalbret (2002) technique for teaching time value of money (TVM) to dyslexic students. Introductory students often report difficulty solving time value of money (TVM) questions due to an inability to correctly identify the variables to be used and the type of problem being solved. This effect is magnified for dyslexic students since the vast majority of TVM questions are word problems and the most oft sighted difficulty associated with dyslexia is reading difficulty. The Jalbert flow chart method is selected because it provided precise definitions and a visual representation of mathematical content both of which are beneficial to dyslexic students. The results of this pilot case study conducted on a matched sample of 10 dyslexic and mainstream students at a large Midwestern University suggests that this modified Jalbert (2002) method is beneficial for dyslexic students.

Keywords: dyslexia, finance, time value of money



## INTRODUCTION

Time value of money (TVM) is arguably the most fundamental component of financial education. For this reason, several different methods have developed over the years for teaching these concepts. Bagamery (1991) developed the “wrist watch method” which breaks down annuity problems into single sum present (PV) or future value (FV) calculations. Forbes (1995) suggests that PV/FV confusion can be eliminated by treating them as foreign currencies and asking students to calculate the effective exchange rate. Eddy and Swanson (1996) suggest that one should avoid mentioning interest rates or introducing the PV and FV formulas or calculations until the students have established an intuitive conceptual framework. Jalbert (2002) advocates the use of a flowchart based instruction to assist students in determining the appropriate problem solving methodology. This research is important because as Bloom (1956) explains, a pedagogy which works well with one audience may fail another. Students differ by ability, learning style and personality type and it is our responsibility to make every effort to insure their success at acquiring basic financial knowledge such as TVM. Despite this vast array of research, no paper to date has examined the impact that dyslexia has on a students’ ability to acquire, understand, interpret or apply TVM concepts. This essay advocates for and investigates the usefulness of a modified version of the Jalbert (2002) technique for teaching time value of money to dyslexic students. The Jalbert flow chart is selected because it provided precise definitions and a visual representation of mathematical content both of which are beneficial to dyslexic students. The results of this pilot case study conducted on a matched sample of 10 dyslexic and mainstream students at a large Midwestern University suggests that this modified Jalbert (2002) method is beneficial for dyslexic students.

## DYSLEXIA: DEFINITION, STATISTICS AND IMPACT

Dyslexia is a specific learning disability that is neurobiological in origin. “It is characterized by difficulties with accurate and/or fluent word recognition and by poor spelling or decoding abilities.” (Lyon, 2003) Estimates of the prevalence of dyslexia conservatively range between 3 and 6% of the general population. The Report of National Working party on Dyslexia from the UK estimates that dyslexia impacts approximately 4% of the general population and between 1.2-1.5% of those who pursue higher education (Singelton, 1999). Also in the UK, Miles (2004) estimates that, while 3% of children are diagnosed with severe cases of dyslexia, an additional 6% may be afflicted with mild cases. In the US 6-7% of the school aged population qualifies for special education as a result of a documented specific learning disability, 85% of those students are classified as being dyslexic (IDA fact sheet, 2008). As noted by Miles (2004) it is difficult to accurately estimate propensity due to differing definitions and severity cutoffs across studies and countries. Also, since the vast majority of research has been conducted within only the past 20-30 years, a large portion of the population may have gone undiagnosed. In fact, Singelton (1999) noted that only 57% of dyslexic students enrolled in higher education knew that they were dyslexic at the time of their enrollment. The International Dyslexia Association estimates that as many as of 15-20% of the worldwide population may be afflicted (IDA fact sheet, 2008).

The vast majority of research over the past 30 years has focused on the impact of dyslexia on an afflicted individual’s ability to learn to read and its continual effects on reading fluency and comprehension over the course of the person’s life (Shaywitz et al., 2008). However, recent

research has begun to highlight the impact that dyslexia can have on mathematical reasoning and education. Several cognitive features of dyslexia which can affect a student's success in mathematics have been identified. Among them are: working and long-term memory retrieval weakness, difficulty memorizing step-by-step procedures, and sequencing difficulties (Kay and Yeo, 2003). It has also been noted that in addition to these issues, a "number of dyslexic learners" may experience visual-spatial and perceptual weaknesses, visual memory issues and left-right difficulty (Kay and Yeo, 2003). As a consequence of these issues, afflicted individuals may exhibit difficulty memorizing number facts (such as multiplication tables) as well as correctly "doing math operations" (IDA fact sheet, 2008). Dyslexia even impacts something as simple as copying a number or problem from one paper to another (transference) due, in part, to the tendency for reversals (Trott, 2003).

Dyslexics tend to have inferior recall of math facts compared to their peers (Butterworth, 2005; Joffe, 1980). This phenomenon most likely stems from long term memory retrieval weakness. Affected individuals tend to have trouble seeing the linkage between steps in a multi-step calculation since they must focus more than their peers on each individual step and thus tend to think of math as a series of "isolated tracks" rather than an interrogated problem solving methodology (Joffe, 1980). Working memory deficits are also common and may result from a lack of the root knowledge of basic math facts and sequencing confusion. Because these students must work through calculations which their peers can simply manually recall, they often lose their way when working through multi-step problems as a consequence of overtaxing their working memories. Sequencing difficulties retard their ability to see patterns in strings of numbers and thus impact even the most basic of mathematical operations. When combined, these issues often cause dyslexics to resort to inefficient counting based strategies to answer basic facts when solving problems (Joffe, 1980). This process expands the number of steps involved in any given calculation, thus offering the dyslexic learner numerous additional chances for error, especially when the impact of their sequencing confusion is considered! Also one must keep in mind that at the college level, math problems are not usually presented as a clear series of numbers with a prescribed operation (+, -, x, /) but rather as "word problems" thus, dyslexic students' reading difficulties can brutally compound their troubles.

## **THE IMPACT OF DYSLEXIA ON A TVM PROBLEM**

The implications of these issues in financial education are enormous and can be illustrated by the following example:

Kelli and Steve plan to retire in 40 years. They anticipate that they will require \$40,000 per year to live in retirement. They believe that they will live in retirement for 45 years. If they make annual payments into their retirement savings account, how much must they save per year in order to meet their goal? Assume a 4.5% annual interest rate. See Figure 1 (Appendix).

The first step most professors would suggest would be to draw out the timeline as shown above. Next one should calculate the present value of the 45 year annuity in year 40, then calculate how much must be saved per year between years 1 and 39 to afford this. This is a difficult problem for most students the first time they are introduced to it, but consider it from the prospective of a dyslexic student. First, it is a "word problem" which requires the student to accurately read to identify the relevant information and intended task. Since reading difficulties are the most pronounced component of dyslexia, this obviously presents a problem. Next the student, faced with sequencing difficulties and possible left-right ambiguity, must draw a time

line. This step may also be impacted by an inability to see the multistage nature of the problem. If the impaired student intends to “solve” the problem manually, his/her working memory may be taxed as a consequence of poor math fact recall and thus they may be more likely to make a math or algebraic error (also heavily influenced by sequencing issues). Even if a financial calculator is used, transference and reversal issues may cause him/her to enter the terms incorrectly!

### **THE JALBERT(2002) METHOD**

Jalbert (2002) develops a comprehensive process to assist students in determining the appropriate problem solving methodology to employ. This flow chart-based system requires students to answer a series of 4 initial questions, then a series of yes or no questions to arrive at the appropriate methodology. The chart is set up to indicate: single sum, perpetuity, growing perpetuity, annuity, uneven cash flow and combination problem calculations. The 4 initial questions are: 1) Is there a series of cash flows? 2) Is the number of cash flows limited? 3) Is there equal spacing between each cash flow? 4) Is the dollar amount of each cash flow equal? Once the problem type is identified via the chart, the students can solve for any of the components via their financial calculators.

### **TEACHING DYSLEXIC STUDENTS**

Multisensory teaching methods for dyslexic students have been advocated and studied for decades beginning with Steevens (1979). These techniques include repeated tactile, auditory and visual manipulations of the same data, at times accomplished simultaneously. In financial education this would involve solving the same problem manually as well as with a financial calculator and Excel.

Crawley and Reines (1996) advocated the use of a four part, “interactive unit” (IU) which provides the instructor with 16 combinations of interactions that can be used to teach elementary mathematics. The IU removes the impact of the student’s disability while still allowing the teacher to address the mathematical concept because of the wide array of presentations. The principle components of this system involve identification of the problem in question, manipulatives (manipulation of a tactile material), and writing and stating the problem verbally. Trott (2003) presents several suggestions for facilitating mathematical learning by dyslexics at the university level. A select list of her suggestions and their applications to financial education follows:

- “Break down a multi-step problem into small, manageable steps.”
  - By clearly identifying the component steps you are acknowledging the “isolated tracks” mentality of the impaired student while still providing an overview of the process.
- “Use colored pens to highlight various aspects of a question.”
  - Colors can be used to distinguish between, PV, FV, PMT,  $i$  and  $N$  when performing basic time value of money calculations. This will help the students to more effectively track the data.
- “Provide flow diagrams or tree diagrams for clarifying procedures.”

- This can help students to see the mathematical “big picture,” Different colors can also be used to further distinguish each step and color families (or hues) can be used to highlight interrelationships.

The effectiveness of these techniques is demonstrated in Trott and Perkin (2005). This case study focuses on a dyslexic student’s struggle to solve an optimization problem using a lagrangian. Each coefficient was written in a different color to “enable him to more clearly identify and separate them.” Then the partial derivatives with respect to each color coded coefficient were calculated below with lagraingian with lines linking them to the initial function creating a tree diagram for the problem.

### **THE MODIFIED JALBERT (2002) TECHNIQUE**

In order to modify the Jalbert (2002) method to meet the requirements of a dyslexic students a few simple modifications were made including color coding and an additional question. The Jalbert (2002) method was developed specifically to appeal to students who benefit from precise definitions and visual aids. The original method instructed students to answer four basic questions to determine which technique to use to solve a TVM problem. A fifth question is added here to facilitate its application to deferred annuities. Each of the questions represents one square on the top of a flow chart. The answers given to each of these questions then determines a student’s progress through the flow chart which ultimately identifies the appropriate technique to solve the problem. The questions and a color coded version of the flow chart are shown in Figure 2 (Appendix).

The hypothesis being tested is that the use of this modified chart during instruction will be a more effective way to teach these concepts to dyslexic students. This technique is visually based as suggested by Hendrickson et al. (2010) and Beacham et al. (2003) . It breaks multi-step problems into small and manageable steps and utilizes a visual framework as advocated by Trott (2003). It also requires the student to manipulate the problem presentation as suggested by Crawley and Reines (1996). Finally, it helps to alleviate the isolated tracks mentality associated with longer math problems as explained by Joffe (1980) by linking the various steps together with a comprehensive system. This system is similar to the Trott and Perkin (2005) largrangian example in that it uses color to highlight the relationships between steps while simultaneously requiring the student to identify each step separately.

Now, let’s apply the modified Jalbert (2002) method to the retirement problem shown above. Consistent with the standard instructional methodology one begins by asking: What am I being asked to solve for? And, what variables am I given?

In this case the question asks how much Kelli and Steve need to save between years 1 and 40 in order to spend \$40,000 per year in retirement. With respect to the variables given, there are two “N’s” in this problem: they will live in retirement for 45 years and they will retire in 40 years. There are also two sets of payments the \$40,000 per year they will live off of in retirement and the payments to savings that they must make between years 1 and 40 to afford their retirement. We are also given the interest rate of 4.5%. Next one must determine what type of problem this is in order to select a problem solving methodology; this is where the modified Jalbert (2002) technique comes into play.

We start with the in the upper left hand corner blue box and work right and down through the flow chart based on the answers to each question:

1. Does my problem have a stream of cash flows (*or just one*)?
  - Yes, there are 2 level streams, one of \$40,000 per year and one for a dollar amount yet to be determined.
2. Does my problem have a limited number of cash flows (*or do they go on forever*)?
  - Yes, the number of cash flows is limited, one lasts for 40 years, the other for 45.
3. Are the cash flows in my problem equally spaced?
  - Yes, there is one cash flow per year.
4. Do all of the cash flows in my problem have equal dollar amounts?
  - Yes, they either receive \$40,000 per year or save a constant a dollar amount yet to be determined
5. Do all of the cash flow in my problem begin in year 1 (*or do some begin at another date*)?
  - No, one set of cash flows don't start until they retire in 40 years. So this must be a deferred annuity problem.

Now that we know that this is a deferred annuity problem, we can set up the timeline. Working backwards through the timeline, one must first figure out how much must be accumulate by year 40 in order to be able to make the 45 planned payments of \$40,000 per year. In other words: calculate the PV of the annuity in year 40. Next, figure out how much you must save per year during years 1-39 to result in this amount by year 40. This process is illustrated in figure 3.

The modified Jalbert (2002) method allows the students to more effectively determine the appropriate problem solving methodology to employ. The direct identification of a deferred annuity is beneficial because it allow the instructor to highlight the appropriate number of compounding periods in each step.

## STUDY DESIGN

The subjects, students at a large Midwestern University, were divided into two groups: dyslexic students (defined as students identified by the university's disabilities services department as having a specific reading disability) and a matched control sample of main-stream students. The matching sample was constructed on the basis of gender, academic standing and, number of finance, accounting and/or economics classes taken. Each group was then further divided into either the treatment or non-treatment groups. The treatment group was taught the TVM skills necessary to solve the two test questions by utilizing the modified Jalbert (2002) technique. The non-treatment group received standard instruction in the same TVM skills consistent with the methodology presented in "Fundamentals of Corporate Finance," Brealey et al. (2007). The effectiveness of the treatment was examined based its impact on students' ability to correctly identify the appropriate problem solving methodology for a set of two TVM problems the first of which was a future value of a single sum. The second question is the "retirement" problem which asks for the present value of a deferred annuity. These problems were selected because they are on "opposite ends" of the chart.

All subjects received approximately 1 hour worth of group specific instruction administered by video PowerPoint which the subjects were able to pause and rewind as desired. The PowerPoint presentation covering: present value of a single sum, present value of a set of uneven cash flows, present value of an annuity and present value of a deferred annuity. Each of these sections presented two examples the first of which showed the development of the problem solving methodology as well as calculation. The second example then offered the subjects the

chance to work through the problem solving methodology on their own before being shown the solution.

## RESULTS AND CONCLUSIONS

Two rounds of fliers for the study were emailed to 130 dyslexic students registered with the Office of Services for Students with Disabilities (SSD) at a large Midwestern university. Additional fliers were posted at the SSD office. The final sample size consists of 10 students: 5 dyslexic and 5 controls. The control sample was selected to match the dyslexic sample as closely as possible on the basis of gender, major, year in school, and the number of finance, economics and accounting courses taken. Both samples consist of: 2 juniors, 2 seniors and 1 graduate student. Two of the dyslexic sample (and their corresponding control matches) are education majors, 2 are liberal arts majors and 1 is an engineering major. Two of participants (and their corresponding matches) have taken economics classes; none have taken any finance or accounting courses. Seven of the participants were female, 3 were male. The dyslexic sample subjects received \$20 compensation; the control subjects received \$10.

Two dyslexic subjects and their matching controls received the treatment instruction. Three of the dyslexic subjects and their matching controls received the standard instruction. There is no specific reason for this 3-2 split (6-4 with controls), that is just how it ended up based on the stacking of the study materials.

The dyslexic sample averaged 1 hour to complete the study; the matched sample averaged 48 minutes. In general the participants who received the treatment instruction spent more time on the study than did those who received the traditional instruction. The dyslexic subjects who received the treatment instruction averaged 51.5 minutes; their matched controls averaged 1 hour and 11 minutes. The dyslexic subjects who received the standard instruction averaged 59.33 minutes, their matched controls averaged 25.5 minutes. On average the treatment group took notes on more slides than the control group did. The dyslexics who received the treatment instruction took notes on an average of 8 slides vs. 6 by their controls. The dyslexics who received standard instruction took notes on 10 slides on average compared to the 1.67 notes taken by their controls.

All but one of the subjects selected the correct methodology for both test questions. Subject 8, a dyslexic female senior liberal arts major given traditional instruction, indicated that both problems were single sum future value calculations. When asked why she made these selections she stated that she was pretty sure that the first one was correct but that she was unable to answer the second question so she simply guessed. She spent 45 minutes listening to the PowerPoint presentation and took notes on 7 slides, both of which are on par with the other participants in her subgroup. After she had completed the study the researcher showed her the treatment slides. She said that she thought that system would have worked better for her. While it is difficult to draw any firm conclusions from this case study given the small sample size, this observation does lend some support to the hypothesis that the use of the modified Jalbert (2002) method will be beneficial to dyslexic students especially since the dyslexic students were better engaged via this method as demonstrated by their increased note taking.

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FIGURES, TABLES AND APPENDICIES

FIGURE 1

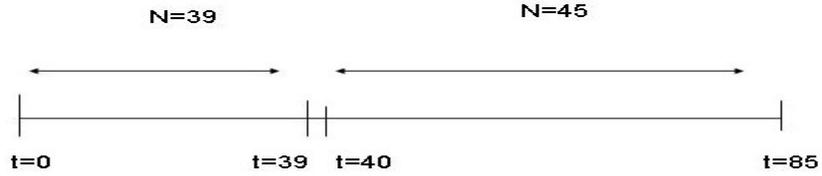


FIGURE 2

Blue Boxes Questions:

1. Does my problem have a stream of cash flows (*or just one*)?
2. Does my problem have a limited number of cash flows (*or do they go on forever*)?
3. Are the cash flows in my problem equally spaced?
4. Do all of the cash flows in my problem have equal dollar amounts?
5. Do all of the cash flow in my problem begin in year 1 (*or do some begin at another date*)?

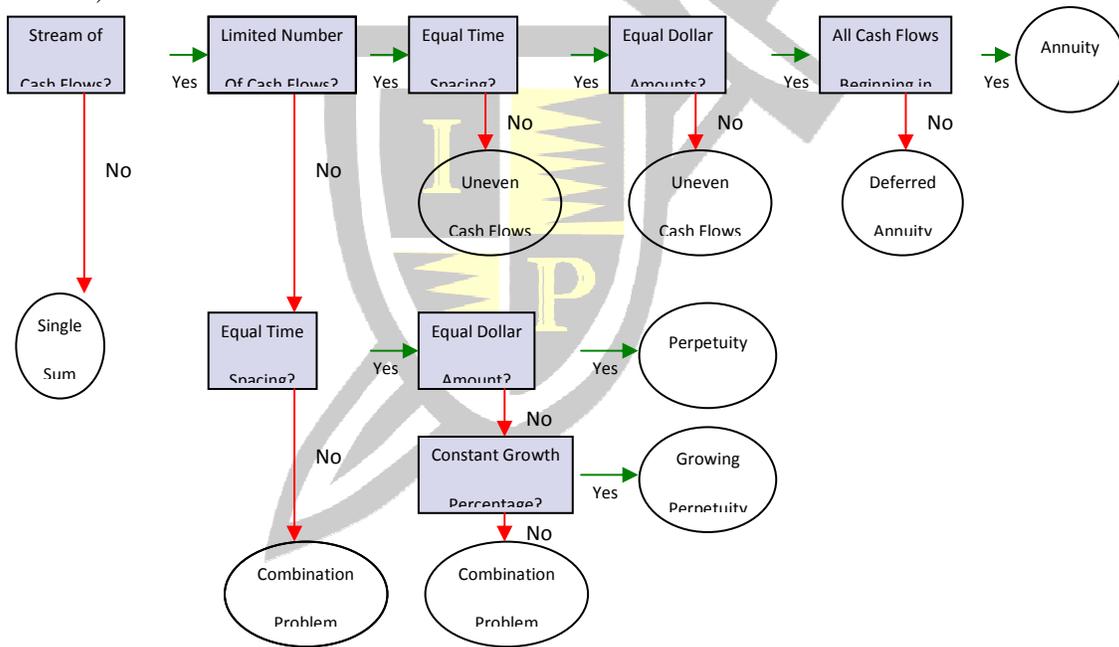
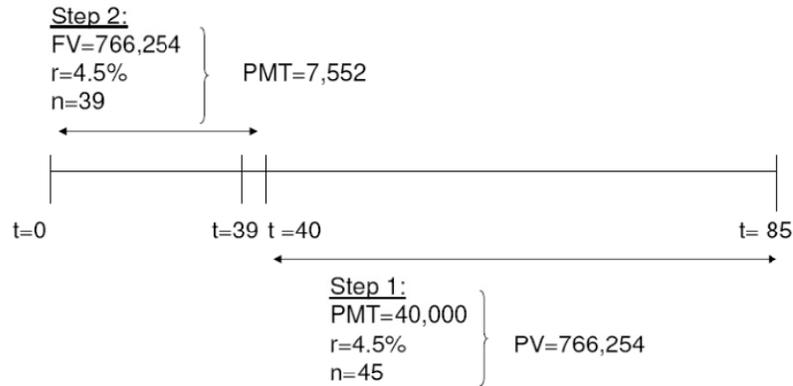


FIGURE 3



APPENDIX 1

These are the treatment PowerPoint slides. The non-treatment slides are textually identical to this set except that it does not utilize the modified Jalbert (2002) flow chart.

**Introduction to Time Value of Money**  
 Method 1

**Topics Covered**

- Compound Interest
- Single Sum Present Values
- Uneven Cash Flows
- Annuities
- Deferred Annuities

**Compound Interest**

- Interest rates are really prices in disguise.
- If \$1 deposited today will be worth \$1.10 next year then the price of \$1 for 1 year is \$ 10
- Tomorrow's amount is known as the future value (FV)
- Today's amount is known as the present value (PV)

**Financial Calculator**

PV: \$      FV: \$      PMT: \$      n:      i: %  
 PV: \$      FV: \$      PMT: \$      n:      i: %  
 (PV) (FV) (PMT) (n) (i)

PV=future value  
 PMT=annual payment  
 i=interest rate  
 n=number of years  
 PV=present value

Link to online calculator used:  
<http://www.zenwealth.com/BusinessFinanceOnline/TV/MTV/CalcWindow.html>

**Questions to Think About**

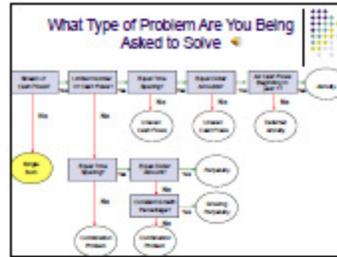
- What variables (PV, FV, n, r, PMT) are you given?
- What variable (PV, FV, n, r, PMT) are you being asked to solve for?
- What type of problem are you being asked to solve?

**What Type of Problem Are You Being Asked to Solve**

Flowchart for problem classification based on given/unknown variables.

### Single Sum Present Value

- Example 1:** You just bought a new computer for \$3,000. The payment terms are 2 years same as cash. If you can earn 8% on your money, how much money should you set aside today in order to make the payment when due in two years?
- What are you being asked to solve for?
  - How much money should you set aside today meaning what is the PV of 3,000 paid out in 2 years?



### Single Sum Present Value

- Example 1:** You just bought a new computer for \$3,000. The payment terms are 2 years same as cash. If you can earn 8% on your money, how much money should you set aside today in order to make the payment in two years?

TVM Solver

FV: 3000    P/Y: 1    C/Y: 1

N: 2    I/Y: 8    PMT: 0

Mode: PV    P1: 1    P2: 2

Result: PV = -2722.09

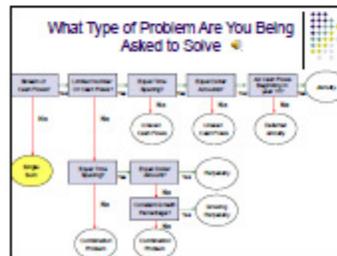
- You should set \$2,722.09 aside today

### Single Sum Present Value

- Example 2:** Jane and John anticipate that their newborn daughters' college tuition will cost \$150,000. How much must they set aside today to insure that they can pay for her education in 18 years if the interest rate is 7%?
- What variables (PV, FV, n, r, PMT) are you given?
- What variable (PV, FV, n, r, PMT) are you being asked to solve for?
- What type of problem are you being asked to solve?

### Single Sum Present Value

- Example 2:** Jane and John anticipate that their newborn daughters' college tuition will cost \$150,000. How much must they set aside today to insure that they can pay for her education in 18 years if the interest rate is 7%?
- What variables (PV, FV, n, r, PMT) are you given?
  - FV=\$150,000
  - n=18 years
  - r=7%
- What variable (PV, FV, n, r, PMT) are you being asked to solve for?
  - How much must they set aside today meaning what is the PV of \$150,000 paid out in 18 years?
- What type of problem are you being asked to solve?



### Single Sum Present Value

• **Example 2:** Jane and John anticipate that their newborn daughter's college tuition will cost \$150,000. How much must they set aside today to insure that they can pay for her education in 10 years if the interest rate is 7%?

Try to solve it:

Pr: € 34867      Rate: 7%

Nr: 10      Period: 10

Pr: € 150000      Rate: 7%

Pr: € 150000      Rate: 7%

• They should set \$44,300 aside today

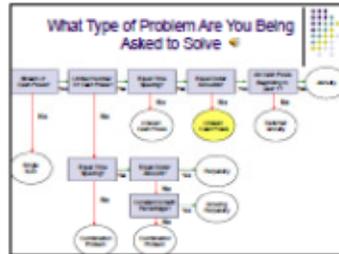
### Uneven Cash Flows

• **Example 1:** An auto dealer offers you the choice to pay \$15,500 cash now, or make three payments: \$5,000 now and payments of \$4,000 next year and \$7,000 the following year. Which do you prefer at an 8% interest rate?

• What are you being asked to solve for?  
• Does the PV of this stream of cash flows exceed \$15,500?

### Uneven Cash Flows

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### Uneven Cash Flows

• **Example 1:** An auto dealer offers you the choice to pay \$15,500 cash now, or make three payments: \$5,000 now and payments of \$4,000 next year and \$7,000 the following year. Which do you prefer at an 8% interest rate?

Try to solve it:

Pr: € 15500      Rate: 8%

Nr: 3      Period: 3

Pr: € 5000      Rate: 8%

Pr: € 4000      Rate: 8%

Pr: € 7000      Rate: 8%

### Uneven Cash Flows

• **Example 1:** An auto dealer offers you the choice to pay \$15,500 cash now, or make three payments: \$5,000 now and payments of \$4,000 next year and \$7,000 the following year. Which do you prefer at an 8% interest rate?

•  $PV = 5,000 + 3,703.70 + 5,348.20 = 14,052.90$   
You should pay the \$15,500 upfront



### Uneven Cash Flows

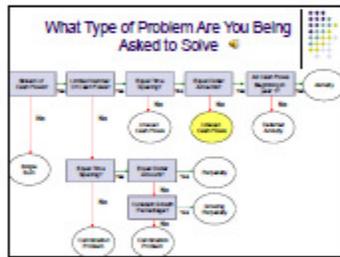
**Example 2:** A factory costs \$400,000. Its projected cash flows for the following 3 years are: \$120,000 in year 1, \$180,000 in year 2, \$300,000 in year 3. Is this a good investment if you plan to sell the factory in 3 years. Assume a 12% annual interest rate.

- What variable (PV, FV, n, i, PMT) are you given?
- What variable (PV, FV, n, i, PMT) are you being asked to solve for?
- What type of problem are you being asked to solve?

### Uneven Cash Flows

**Example 2:** A factory costs \$400,000. Its projected cash flows for the following 3 years are: \$120,000 in year 1, \$180,000 in year 2, \$300,000 in year 3. Is this a good investment if you plan to sell the factory in 3 years. Assume a 12% annual interest rate.

- What variables (PV, FV, n, i, PMT) are you given?
  - PV = -400,000; FV = 0; n = 3
  - i = 12%
- What variable (PV, FV, n, i, PMT) are you being asked to solve for?
  - Does the PV of the anticipated cash flows exceed the purchase price of \$400,000?
- What type of problem are you being asked to solve?



### Uneven Cash Flows

**Example 2:** A factory costs \$400,000. Its projected cash flows for the following 3 years are: \$120,000 in year 1, \$180,000 in year 2, \$300,000 in year 3. Is this a good investment if you plan to sell the factory in 3 years. Assume a 12% annual interest rate.

### Uneven Cash Flows

**Example 2:** A factory costs \$400,000. Its projected cash flows for the following 3 years are: \$120,000 in year 1, \$180,000 in year 2, \$300,000 in year 3. Is this a good investment if you plan to sell the factory in 3 years. Assume a 12% annual interest rate.

**PV Calculator**

PV:  Rate:  %  
 FV:  Period:   
 PMT:  Annual:

**FV Calculator**

PV:  Rate:  %  
 FV:  Period:   
 PMT:  Annual:

### Uneven Cash Flows

**Example 2:** A factory costs \$400,000. Its projected cash flows for the following 3 years are: \$120,000 in year 1, \$180,000 in year 2, \$300,000 in year 3. Is this a good investment if you plan to sell the factory in 3 years. Assume a 12% annual interest rate.

**PV Calculator**

PV:  Rate:  %  
 FV:  Period:   
 PMT:  Annual:



### Uneven Cash Flows

**Example 1:** A factory costs \$400,000. Its projected cash flow for the following 3 years are \$120,000 in year 1, \$180,000 in year 2, \$300,000 in year 3. Is this a good investment if you plan to use the factory at 12% annual interest rate?

$PV = 107,143 + 140,495 + 213,334 = 461,172$

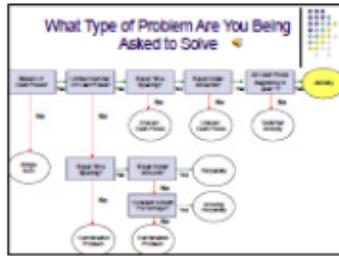
• This is a good investment because the PV of the anticipated cash flows exceeds the purchase price.

### Annuities

**Example 1:** The local furniture store is advertising the following sale. "Walk away today with a brand new leather couch! No money down and payments as low as \$450 per year for the next 5 years." The regular purchase price of the couch is \$1,999. Is this really a sale if the interest rate charged is 4.5%?

• What are you being asked to solve for?
 

- Is the regular price higher than the sales price meaning, is the PV of the annual cash flows of \$450 less than \$1,999?



### Annuities

**Example 1:** The local furniture store is advertising the following sale. "Walk away today with a brand new leather couch! No money down and payments as low as \$450 per year for the next 5 years." The regular purchase price of the couch is \$1,999. Is this really a sale if the interest rate charged is 4.5%?

### Annuities

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• YES, this is a sale, the offer price is less than the regular price.

### Annuities

**Example 2:** Kyle just won a "win for life" lottery which will pay him \$250,000 per year for the rest of his life. An investor offers to purchase this annuity from Kyle for \$4,000,000 today. If Kyle is currently 22 and is expected to live to 85 is this a good deal for him? Assume a 5% annual interest rate.

• What variable (PV, FV, i, n, PMT) are you given?

• What variable (PV, FV, i, n, PMT) are you being asked to solve for?

• What type of problem are you being asked to solve?



### Annuities

- Example 2:** Kyle just won a "lot for life" lottery which will pay him \$25,000 per year for the rest of his life. An investor offers to purchase this annuity from Kyle for \$4,000,000 today. If Kyle is currently 22 and is expected to live to 85 is this a good deal for him? Assume a 5% annual interest rate.
  - What variables ( $PV$ ,  $FV$ ,  $i$ ,  $n$ ,  $PMT$ ) are you given?
    - $PMT = \$25,000$
    - $i = 5\%$
    - $n = 63$  (85-22)
  - What variable ( $PV$ ,  $FV$ ,  $i$ ,  $n$ ,  $PMT$ ) are you being asked to solve for?
    - Solve for  $PV$  if the entrepreneur will then compare the offer to \$4,000,000.
  - What type of problem are you being asked to solve?



### Annuities

- Example 2:** Kyle just won a "lot for life" lottery which will pay him \$25,000 per year for the rest of his life. An investor offers to purchase this annuity from Kyle for \$4,000,000 today. If Kyle is currently 22 and is expected to live to 85 is this a good deal for him? Assume a 5% annual interest rate.

### Annuities

- Example 2:** Kyle just won a "lot for life" lottery which will pay him \$25,000 per year for the rest of his life. An investor offers to purchase this annuity from Kyle for \$4,000,000 today. If Kyle is currently 22 and is expected to live to 85 is this a good deal for him? Assume a 5% annual interest rate.
 

$PV = \$4,000,000$      $i = 5\%$   
 $PMT = \$25,000$      $n = 63$   
 $FV = 0$      $n = 63$      $i = 5\%$      $PMT = \$25,000$
- No, this is not a good investment because the PV of the anticipated cash flows is less than the offer price.

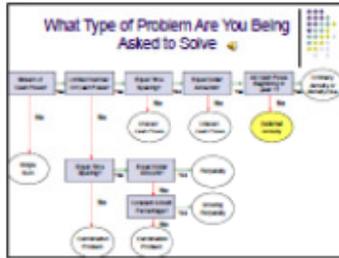
### Deferred Annuity

- Deferred Annuity:** An annuity where the first payment is delayed by a pre-specified number of years.
  - Remember today is year 0

### Deferred Annuity

- Example 1:** A couple plan to retire in 50 years. They expect that they will require \$40,000 per year to live in retirement. They anticipate living in retirement for 30 years. If they earn a 3.5% interest rate, how much must they set aside now in order to meet their goal?
  - What are you being asked to solve for?
    - How much should they have in year 0?





### Deferred Annuity

- Example 1:** A couple plan to retire in 30 years. They expect that they will require \$40,000 per year to live in retirement. They anticipate being in retirement for 30 years. If they earn a 3.5% interest rate, how much must they set aside now in order to meet their goal?

### Deferred Annuity

- Step 1:** Figure out how much they must accumulate by year 30 in order to be able to make the 30 planned payments of \$40,000 per year
- Step 2:** Figure out how much they must have today in order to have this amount in year 30

### Deferred Annuity

**Step 1**

TVA Calculator

PV: 470000    Rate: 3.5%    PMT: 40000    FV: 0    N: 30

PV: 470000    Rate: 3.5%    PMT: 40000    FV: 0    N: 30

**Step 2**

TVA Calculator

PV: 470000    Rate: 3.5%    PMT: 40000    FV: 0    N: 30

PV: 470000    Rate: 3.5%    PMT: 40000    FV: 0    N: 30

### Deferred Annuity

- Example 1:** A couple plan to retire in 30 years. They expect that they will require \$40,000 per year to live in retirement. They anticipate being in retirement for 30 years. If they earn a 3.5% interest rate, how much must they save per year to meet their goal?

Step 2: PV = \$136,336.73

Step 1: PV = \$136,336.73

**They must save \$136,336.73 today**

### Deferred Annuity

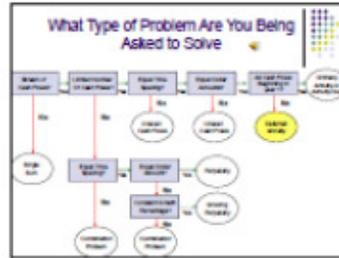
- Example 2:** Jen and Andy are now 28 years old and plan to retire when they reach age 60. They wish to retire on \$65,000 per year. If they both expect to live to age 90, how much should they have today in an account earning 6.5% annual interest to achieve their retirement objective?

- What variables (PV, FV, i, n, PMT) are you given?
- What variable (PV, FV, i, n, PMT) are you being asked to solve for?
- What type of problem are you being asked to solve?



### Deferred Annuity

- Example 1:** Jim and Andy are now 28 years old and plan to retire when they reach age 60. They want to retire on \$80,000 per year. If they both expect to live to age 90, how much should they have today in an account earning 5.5% annual interest to achieve their retirement objective?
  - What variable (PV, FV, i, n, PMT) are you asked?
  - What retirement objective?
    - Retirement \$80,000
    - at 5.5%
- What variable (PV, FV, i, n, PMT) are you being asked to solve for?
  - How much should they have today?
- What type of problem are you being asked to solve?



### Deferred Annuity

- Example 2:** Jim and Andy are now 28 years old and plan to retire when they reach age 60. They want to retire on \$80,000 per year. If they both expect to live to age 90, how much should they be saving per year in an account earning 5.5% annual interest to achieve their retirement objective?

### Deferred Annuity

- Example 2:** Figure out how much they must accumulate by age 60 in order to be able to make the planned payments of \$80,000 per year until age 90.
- Example 1:** Figure out how much they must set aside today in order to save this amount in at age 60.

### Deferred Annuity

**Step 1**      **Step 2**

**They must save \$120,486.68 today**

### Deferred Annuity

- Example 1:** Jim and Andy are now 28 years old and plan to retire when they reach age 60. They want to retire on \$80,000 per year. If they both expect to live to age 90, how much should they be saving per year in an account earning 5.5% annual interest to achieve their retirement objective?

**Step 2: PV = 120,486.68**

**Step 1: PMT = \$128.08**

**They must save \$ 9128.08 per year**



## APPENDIX 2

### Time Value of Money Test Questions

At the conclusion of the PowerPoint presentation the subjects were instructed to review the slides prior to taking this 2 question test. They were told that they would not be required to solve any problems mathematically, but would instead be asked to identify the appropriate problem solving methodology.

1. Jane invested \$500 dollars into a bank account earning 3% interest annually, what will the value of this account be in 20 years, assuming she makes no additional contributions or withdraws?
  - A) Single Sum Future Value
  - B) Uneven Cash Flow
  - C) Deferred Annuity
  - D) Annuity

Answer: A

2. Kelli and Steve plan to retire in 40 years. They anticipate that they will require \$40,000 per year to live in retirement. They believe that they will live in retirement for 45 years. If they make annual payments into their retirement savings account, how much must they save per year in order to meet their goal? Assume a 4.5% annual interest rate.
  - A) Single Sum Future Value
  - B) Uneven Cash Flow
  - C) Deferred Annuity
  - D) Annuity

Answer: C

