

Nonlinear mean-reversion in the real exchange rate: evidence from a panel of OECD countries

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Abstract

We examine the presence of mean-reversion in the real exchange rates of a panel of 20 OECD using the standard Dickey-Fuller test and smooth transition autoregression (STAR) framework. Our results provide strong support for the long run empirical validity of the purchasing power parity in most all major industrial countries. Our findings attribute the nonlinear stationarity of the real exchange rates to certain nonlinearities in the short run adjustment paths of these rates. Such nonlinearities in the adjustment process may arise by more aggressive market reaction and government intervention when its departure from equilibrium is more drastic.

Keywords: time series, exchange rates, market efficiency



Introduction

Since the path-breaking work of Nelson and Plosser (1982), it is widely believed that most macroeconomic and financial time series are characterized by linear unit root processes. As pointed out by Campbell and Perron (1991), this finding has important implications for both theoretical and empirical research. In particular, the presence of unit roots indicates that the short run departures of the underlying variables from their long run equilibrium values are fairly persistent and irreversible, a finding which is at odds with many of the existing theories of economic and financial behavior, where such departures are assumed to be short-lived and self-correcting. Given the unfamiliar implications of unit root processes, there has been a three-pronged scholarly response. First, having accepted the basic premise that most time series data do contain unit roots, some researchers have attempted to rationalize this fact by new theoretical models. Examples include the development of the real business cycle theory (Kydland and Prescott, 1982) to explain the presence of unit roots in real outputs, and the efficient markets hypothesis (Fama, 1991) to justify the random walk character of stock prices. Second, another strand of research has attempted to refute the Nelson-Plosser findings of unit roots in major time series by using more powerful econometric techniques, such as the Bayesian (Schotman and van Dijk, 1991) and panel data unit root tests (Frankel and Rose, 1996). Finally, other researchers have tried to attribute the failure to reject the null of unit roots in many macroeconomic and financial variables to the use of inadequate alternative hypotheses. Instead of testing for unit roots against the alternative of stationarity around a linear trend, as is commonly done in standard unit root tests, it is recommended that these tests be conducted against the alternative of stationarity around a nonlinear trend. Examples of these nonlinear alternative hypotheses, often justified by the presence of nonlinearities in the underlying adjustment processes, include the use of threshold autoregression (Balke and Fomby, 1997), shifted intercepts and broken trends (Perron, 1989; Zivot and Andrews, 1992), autoregressions subject to ceilings and floors (Pesaran and Potter, 1997), asymmetric autoregression (Enders and Granger, 1998), and smooth transition autoregression (Terasvitra, 1994; Kapetanios, Shin, and Snell, 2002).

Among the above nonlinear alternative hypotheses, the smooth transition autoregression (STAR) has received considerable attention in recent years (see van Dijk, Terasvitra, and Franses, 2002, for an excellent survey). Like many other nonlinear approaches to time series modeling, the STAR approach is based on the existence of a number of distinct regimes, each becoming operational at a different time in response to a different set of circumstances. Unlike other approaches, however, where the transition from one regime to another is usually sudden and abrupt, the STAR model assumes these transitions to be gradual and smooth. In other words, by allowing the existence of middle ground regimes, the STAR model provides a potentially more flexible framework to capture the time series behavior of many macroeconomic and financial time series.

Given the preceding discussion, this paper attempts to illustrate the use of the STAR framework in analyzing the time series behavior of the real exchange rate in 20 OECD countries over the 1970-2011 period. More specifically, the STAR model is used as the alternative nonlinear stationarity hypothesis for testing the null hypothesis of unit roots in the real exchange rates. Given the importance of the real exchange rate as a key macroeconomic variable, there already exists a voluminous empirical literature on its time series behavior (see Rogoff, 1996, and Sarno and Taylor, 2002, and Taylor, 2006, for excellent surveys of the literature). The main purpose of most of this literature is to establish whether the real exchange rate is characterized

by the presence of unit roots during the recent float, a condition unfavorable to the long run mean reversion of the real exchange rates and, hence, to the empirical validity of the long run purchasing power parity (PPP). In particular, Roll (1979), Adler and Lehmann (1983), Taylor (1988), Mark (1990), and Fraser et al. (1991), among others, have found evidence of unit roots in the real exchange rate for a large number of industrial and developing countries. Since these findings are inconsistent with the long run PPP, other researchers using a host of other approaches, including fractional integration, multivariate cointegration, panel data unit root tests, and longer sample periods, have uncovered evidence more favorable to the stationarity of the real exchange rates (Taylor and McMahon, 1988; Kugler and Lenz, 1993; Frankel and Rose, 1996; Lothian and Taylor, 1996; MacDonald, 1996; Baum et al., 1999; Flores, et al., 1999; Cheung and Lai, 2000; Taylor, 2002). The latter attempts to salvage the long run PPP, however, suffer from a number of drawbacks. Most importantly, they are all based on essentially linear processes that ignore such nonlinear effects as transactions costs, variable speeds of adjustment, regime shifts, and monopolistic pricing effects (e.g., Dumas, 1992; Ohanian and Stockman, 1997; Chari et al. 2000; Baum et al. 2001; Sarno et al., 2004). In addition, by relying on longer sample periods, some of these studies include periods of fixed exchange rates which have been shown to bias the results in favor of the stationarity of the real exchange rates (Rogoff, 1996; Engel, 2000). Finally, incorporating the foregoing nonlinear effects into the real exchange rate behavior, a number of recent studies have relied on the STAR model to derive evidence in support of the long run PPP (Taylor, et al., 2001; Chortareas et al., 2002; Lathinen, 2006; Sekioua and Karanasos, 2006).

This paper advances the evidence on the empirical validity of the long run PPP by testing for the nonstationarity of the real exchange rate against a nonlinear (STAR) alternative for 20 major OECD countries during the recent period of float, 1970-2011. Thus our study covers a larger number of major industrial countries and for a longer span of time than earlier studies. In addition, instead of using bilateral exchange rates, as is common in most earlier empirical work, we rely on the trade-weighted real exchange rates, as reported by the OECD.

The rest of the paper is organized as follows. Section II discusses the econometric methodology employed. Section III presents the empirical results. Section IV concludes.

Methodology

As stated earlier, the main objective of this paper is to test the null hypothesis of unit roots in the real exchange rates of a sample of OECD countries against the alternative of stationarity within a smooth transition autoregression (STAR) framework. A univariate STAR process in a mean-zero (i.e., detrended) stochastic process y_t can be expressed as:

$$y_t = \beta y_{t-1} + \gamma y_{t-1} \Theta(\theta; y_{t-d}) + \varepsilon_t, t = 1, \dots, T, \quad (1)$$

where $\varepsilon_t \approx iid(0, \sigma^2)$, β and γ are unknown parameters, representing two alternative autoregressive regimes, and $\Theta(\theta; y_{t-d})$ is the transition function, with θ = speed of trend-reversion, and d = delay parameter. In addition, the transition function is assumed to take the following exponential form:

$$\Theta(\theta; y_{t-d}) = 1 - \exp(-\theta y_{t-d}^2), \quad (2)$$

where it is assumed that $\theta \geq 0$ and $d \geq 1$. Clearly, the transition function can adopt any value between zero and one. Combining (1) and (2), we obtain:

$$y_t = \beta y_{t-1} + \gamma y_{t-1} [1 - \exp(-\theta y_{t-d}^2)] + \varepsilon_t, \quad (3)$$

which can alternatively be rewritten as:

$$\Delta y_t = \phi y_{t-1} + \gamma y_{t-1} [1 - \exp(-\theta y_{t-d}^2)] + \varepsilon_t, \quad (4)$$

where $\phi = \beta - 1$. Clearly, if $\phi = \theta = 0$, y_t will have a unit root as one possible autoregressive regime, and if $\phi = 0$ and $\theta > 0$, y_t will follow a nonlinear but stationary process as an alternative regime, assuming that $-2 < \gamma < 0$. Furthermore, the delay parameter d is chosen to maximize the goodness of fit of (4) over $\{1, 2, \dots, d_{\max}\}$, where d_{\max} is determined by using one of the usual lag selection procedures.

If, following Kapetanios, Shin, and Snell (2002, henceforth, KSS), the condition $\phi = 0$ is imposed, (4) can be rewritten as:

$$\Delta y_t = \gamma y_{t-1} [1 - \exp(-\theta y_{t-d}^2)] + \varepsilon_t. \quad (5)$$

Now, the null hypothesis of a unit root against the alternative of a nonlinear STAR stationarity can be expressed as:

$$H_0 : \theta = 0, \quad (6)$$

$$H_1 : \theta > 0. \quad (7)$$

Since, under the null hypothesis, the nuisance parameter γ cannot be identified (Davies, 1987), the paper follows Luukkonen, Saikkonen, and Terasvitra (1988) and derives a t test by approximating (5) by a first order Taylor expansion (with lagged values of the first differences of y_t added to whiten the error process a la Dickey and Fuller, 1979):

$$\Delta y_t = \sum_{j=1}^p \rho_j \Delta y_{t-j} + \delta y_{t-1} y_{t-d}^2 + e_t \quad (8)$$

Thus, the null hypothesis can be tested as a t test of $\delta = 0$, against the alternative of $\delta < 0$, by using the following statistic:

$$t_{NL} = \delta / s.e.(\delta), \quad (9)$$

using the critical values tabulated by KSS. KSS also suggest an alternative joint F test of $\phi = \delta = 0$ in the following:

$$\Delta y_t = \sum_{j=1}^p \rho_j \Delta y_{t-j} + \phi y_{t-1} + \delta y_{t-1} y_{t-d}^2 + e_t, \quad (10)$$

based on the critical values provided by Enders and Granger (1998).

Empirical Results

This section presents the empirical results of testing for the presence of unit roots in the real exchange rates of our sample countries, using the methodology discussed in the preceding section. The data, which are taken from the OECD files of the RATS software package, are quarterly, calculated as the logs of the real exchange rates, and cover the 1970:1-2011:1 period.

As a first step in the analysis of the time series properties of the real exchange rates, this section conducts the standard Dickey-Fuller unit root tests of these rates against the alternative hypotheses that they are stationary around a constant (since a visual inspection of the real exchange rates indicated no clear trend for any of the countries in the sample, no time trends were included in the Dickey-Fuller tests). As is well known, the implementation of the Dickey-Fuller test requires the whitening of the error terms associated with the auxiliary equations of

these tests by adding an appropriate number of lags of the first differences of the underlying variables to these equations. To establish the appropriate lag length for each of the sample countries, the Akaike information criterion (Akaike, 1973) is used. The Dickey-Fuller unit root test results are given in Table 1. As seen from the table, the null of a unit root cannot be rejected for seventeen of the twenty sample countries (exceptions are Germany, Netherlands, and Norway), indicating the absence of mean-reversion for the real exchange rate in an overwhelming majority of the OECD countries. This finding, taken at face value, offers unfavorable evidence about the empirical validity of the long run PPP for OECD countries.

Having established the random walk behavior of the real exchange rates within the standard Dickey-Fuller framework, this section now proceeds to examine the time series properties of these rates within a STAR model. As stated in the preceding section, the STAR model tests for the presence of unit roots in the real exchange rates against the alternative hypothesis that these rates are stationary within a smooth regime-switching framework.

As also seen from the previous section, the implementation the STAR approach requires tests of significance of certain estimated coefficients in the auxiliary equations (8) and (10). Specifically, this involves a t test of significance of δ in the following equation:

$$\Delta y_t = \sum_{j=1}^p \rho_j \Delta y_{t-j} + \delta y_{t-1} y_{t-d}^2 + e_t \quad (11)$$

See Table 1 (Appendix).

Or, alternatively, an F test of joint significance of ϕ and δ in the following equation:

$$\Delta y_t = \sum_{j=1}^p \rho_j \Delta y_{t-j} + \phi y_{t-1} + \delta y_{t-1} y_{t-d}^2 + e_t, \quad (12)$$

where the numbers of the lags used in the above equations are the same as those previously selected by the Akaike method for the Dickey-Fuller tests. The estimation of the above equations, however, also requires the selection of an appropriate value for d , the delay parameter. To this end, and for each of the sample countries, each of the above equations is first estimated for all values of $1 \leq d \leq d_{\max}$, where d_{\max} represents the optimal lag length previously selected by the Akaike method. Next, the value of d with the best fit, i.e., with the lowest significant p-value, is selected as the optimal delay parameter to be used in the estimation of the corresponding country equations. These estimated equations are then used to conduct the STAR significance tests. The results of these tests are reported in Table 2. It can be seen from the table that, based on both sets of test results, the STAR approach rejects the null of unit roots for sixteen of the twenty countries in the sample in favor of stationarity within a nonlinear STAR framework (the only exceptions being Australia, Greece, Italy, and Portugal where nonlinear stationarity is rejected by the t test but not the F test). These results indicate that for almost all the countries in the sample, there is significant evidence that the real exchange rate has a tendency to revert to its long term mean, with any short term departure from the long term mean being transitory and short-lived. Thus, these results are clearly consistent with similar findings in the literature, referred to earlier in the paper, which document the long run mean-reversion of the exchange rates in major industrial countries. These results, however, are clearly at odds with the standard Dickey-Fuller test results, which, as we have seen earlier, tend to support the random walk character of the real exchange rates. See Table 2 (Appendix).

As a final word, there is a need to justify the nonlinear stationarity of the real exchange rates for almost all the OECD countries. Since the autoregressive equations (8) and (10) above are essentially short run adjustment paths of the real exchange rates towards their long run equilibrium values, the nonlinear stationarity of the real exchange rates can thus be interpreted as the nonlinear adjustment behavior of these rates. More specifically, the STAR model assumes that the speed of adjustment is a function of the size of the deviation of the actual real exchange rates from their equilibrium values. This situation can arise mostly in connection with the prevailing balance of payments reaction functions, where the exchange rates, whether through market forces or government policies, are expected to be triggered more strongly in response to a larger disequilibrium in the exchange rates. For small deviations from the equilibrium values, market reactions or policy responses are usually muted, as such small departures may be deemed too limited and transitory to warrant stronger response. However, for more dramatic departures of the exchange rates from their historical means, the extent of market reaction and government intervention may be more extensive, with the adjustment of the real exchange rates to their equilibrium values taking place at a much faster rate.

Conclusion

This paper has shown that based on the standard Dickey-Fuller unit root test, which is a test of stationarity around a constant, the real exchange rates of a large panel of OECD countries follow random walks and are, thus, largely unfavorable to the long run PPP. The paper, however, has also shown that most of these exchange rates are stationary within a nonlinear STAR framework, which renders them consistent with the empirical validity of the PPP in the long run. Finally, the paper has attributed the nonlinear stationarity of the real exchange rates to certain nonlinearities in the short run adjustment paths of these rates. Such nonlinearities in the adjustment process can arise by more aggressive market reaction and government intervention when its departure from equilibrium is more drastic.

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Appendix

Table 1
Unit Root Test Results
(Dickey and Fuller, 1979)

Country	Lags	t test	Country	Lags	t test
Australia	8	-1.42	Italy	3	-2.76
Austria	5	-2.80	Japan	3	-2.44
Belgium	1	-2.13	Netherlands	1	-3.08*
Canada	3	-1.81	Norway	1	-3.25*
Denmark	9	-2.16	Portugal	4	-1.76
Finland	1	-1.47	Spain	1	-2.51
France	10	-2.59	Sweden	1	-1.74
Germany	3	-3.10*	Turkey	3	-1.59
Greece	6	-1.36	UK	9	-2.13
Ireland	1	-1.65	US	4	-2.87

* Indicates significant at the 5 percent level.

Table 2
STAR Test Results
(Kapetanios, Shin, and Snell , 2002)

Country	d for t and (F) tests	t statistic	F statistic	Country	d for t and (F) tests	t statistic	F statistic
Australia	4(3)	-2.68	12.26*	Italy	1(12)	-0.51	42.98*
Austria	10(5)	-5.73*	96.79*	Japan	7(4)	-3.60*	62.16*
Belgium	10(2)	-24.50*	328.50*	Netherlands	9(2)	-24.44*	100.91*
Canada	1(10)	-3.74*	6.09*	Norway	12(12)	-27.32*	473.45*
Denmark	8(1)	-5.96*	19.31*	Portugal	1(2)	-2.80	9.67*
Finland	9(9)	-3.29*	270.42*	Spain	2(6)	-4.11*	58.99*
France	10(12)	-8.31*	141.92*	Sweden	1(1)	-7.90*	548.64*
Germany	1(1)	-15.09*	187.50*	Turkey	1(5)	-5.93*	425.69*
Greece	9(11)	-2.28	10.62*	UK	5(4)	-8.50*	54.15*
Ireland	7(9)	-3.78*	23.25*	US	12(12)	-11.73*	431.94*

*Indicates significant at the 5 percent level.